Pearson Edexcel

Mark Scheme (Results)

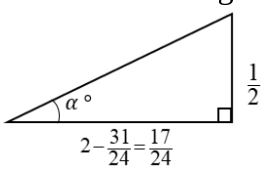
January 2024

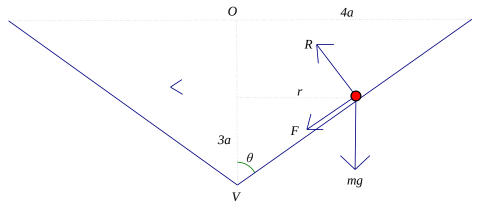
Pearson Edexcel International Advanced Level in Mechanics M3 (WME03) Paper 01

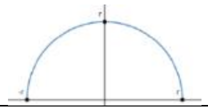
1a	$-\frac{mgR^2}{2x^2}=mv\frac{\mathrm{d}v}{\mathrm{d}x}$	M1	Form differential equation in v and x only. Need to see $\frac{\mathrm{d}v}{\mathrm{d}x}$ or $v\frac{\mathrm{d}v}{\mathrm{d}x}$ Cannot get this mark using t. Allow with both m's cancelled. Condone sign error.
	$-\frac{gR^2}{2}\int\frac{1}{x^2}\mathrm{d}x=\int v\mathrm{d}v$	M1	Separate variables correctly and integrate at least one side. Cannot get this mark using t. Condone sign error.
	$v^2=\frac{gR^2}{x}+C$ *	A1*	Obtain given answer from correct work. Must include at least one line of working between integral and final answer. Correct signs seen throughout working. Condone $\frac{v^2}{2}=\frac{gR^2}{2x}+C$ followed by $v^2=\frac{gR^2}{x}+C$ Note: If the first line of working is $\frac{1}{2}v^2=-\int\frac{gR^2}{2x^2}dx$ followed by integration of RHS, this scores M0M1A0*
ALT1 (a)	$\frac{1}{2}mu^2-\frac{1}{2}mv^2=\int\frac{gmR^2}{x^2}\mathrm{d}x$	M1	Form an energy equation with 2 KE terms and the integral of the variable force. Condone sign errors.
	$\frac{1}{2}mu^2-\frac{1}{2}mv^2=-\frac{gmR^2}{x}+A$	M1	Integrate the force wrt x. Condone sign errors.
	$v^2=\frac{gR^2}{x}+C$ *	A1*	Obtain given answer from correct work. Must include at least one line of working and correct signs seen throughout working.
		[3]	
b	$x=3R, v^2=3gR$	M1	Use initial conditions to evaluate C in the given answer.
	$\Rightarrow C=3gR-\frac{gR^2}{3R}\left(=\frac{8gR}{3}\right)$	A1	Or equivalent
	$x=R\Rightarrow v=\sqrt{\frac{11gR}{3}}$	A1	Accept $\frac{\sqrt{33gR}}{3}$ Answer must be in terms of g and R
		[3]	
ALT1 (b)	Use of definite integral instead of finding + C		
	$\left[v^2\right]_{\sqrt{3gR}}^{v}=\left[\frac{gR^2}{x}\right]_{3R}^{R}$	M1	Use initial conditions in a definite integral.

	$v^2 - 3gR = \frac{gR^2}{R} - \frac{gR^2}{3R}$	A1	Or equivalent
	$v = \sqrt{\frac{11gR}{3}}$	A1	Accept $\frac{\sqrt{33gR}}{3}$ Answer must be in terms of g and R
		(6)	

2a	Change in GPE $= mg \times 1.3l \sin\theta \left(= 0.5lmg\right)$	M1	Condone sin / cos confusion
	$\text{EPE} = \dfrac{\lambda l^2}{2l}$ **or** $\text{EPE} = \dfrac{\lambda\left(0.3l\right)^2}{2l}$	B1	One correct term for EPE
	Energy equation *B* to *A*	M1	Dimensionally correct with all the required terms. Condone sign errors and sin / cos confusion
	$\dfrac{\lambda l^2}{2l} - \dfrac{\lambda\left(0.3l\right)^2}{2l} = 0.5lmg$	A1	Correct unsimplified equation
	$\Rightarrow \lambda = mg\dfrac{1}{1-0.09} = \dfrac{100}{91}mg$	A1*	Obtain given answer from correct working. Must see evidence of simplification.
		[5]	
2b	Equation of motion	M1	Dimensionally correct with all the required terms. Condone sign errors and sin/cos confusion.
	$T - mg\sin\theta = ma$	A1	Correct unsimplified equation
	$\dfrac{\lambda \times l}{l} - mg\sin\theta = ma$ $\left(\dfrac{100}{91}mg - \dfrac{5}{13}mg = ma\right)$	A1	Correct unsimplified equation with HL used to replace *T*
	$a = \dfrac{5}{7}g$	A1	Accept $0.71g$ or better. If *g* = 9.8 is used, accept 7.
	Note: If *g* = 9.81 is used then penalise once per complete question. SHM equations can only be used if the motion is proven to be SHM first.		
		[4]	
		(9)	

3a	Moment of S about the y-axis	M1	Use of formula $(\pi)(\rho)\int xy^2\,\mathrm{d}x$ No need to see the correct limits here. The curve equation must be substituted correctly and an attempt to integrate seen (at least one term must have a power of x raised by 1) Note the correct expression for integrating is $x\left(\frac{1}{4}x(3-x)\right)^2=\frac{1}{16}x^3(3-x)^2$ $=\frac{1}{16}\left(9x^3-6x^4+x^5\right)$
	$=(\pi)(\rho)\frac{1}{16}\left[\frac{9}{4}x^4-\frac{6}{5}x^5+\frac{1}{6}x^6\right]$	A1	Correct integrated expression.
	$=\frac{31}{60}(\pi)(\rho)$	A1	Correct use of correct limits (0 and 2). No need to see a line of working showing substitution of limits. However, must see $\frac{31}{60}$ or equivalent numerical evaluation of integral.
	$\bar{x}=\dfrac{\frac{31}{60}(\pi)(\rho)}{\frac{2}{5}(\pi)(\rho)}$	M1	Complete method to find the distance. Formula must be the right way up $\bar{x}=\dfrac{(\pi)(\rho)\int xy^2\,\mathrm{d}x}{M}$ Must have consistent use of π and of ρ.
	$=\frac{31}{24}$ *	A1*	Obtain given answer from correct working
		[5]	
3b	Correct use of trig  	M1	Correct trig ratio to find a relevant angle, $\alpha°$ or $(90-\alpha)°$ Must use curve equation with x = 2 and $\left(2-\frac{31}{24}\right)$
	$\tan\alpha°=\frac{1}{2}\div\frac{17}{24}\left(=\frac{12}{17}\right)$	A1	Or equivalent. Condone reciprocal.
	$\alpha=35$	A1	2 sf or better (35.2175…) A0 for use of radians.
		[3]	
		(8)	

4			If angle is between incline and vertical then $\sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}$
	Resolve vertically	M1	Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion.
	$R\sin\theta = mg + F\cos\theta$	A1 A1	Unsimplified equation with at most one error. Correct unsimplified equation
	Equation for horizontal motion	M1	Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion. Accept any form of acceleration for the method mark only.
	$R\cos\theta + F\sin\theta = mr\omega^2$	A1 A1	Unsimplified equation with at most one error. Direction of F consistent with vertical resolution. Incorrect form of acceleration is one error. Correct unsimplified equation
	Use of $F = \mu R$	M1	Used, not just quoted. $F = \frac{1}{4}R$
	Substitute for trig and solve for max ω	DM1	Dependent on all preceding M marks. If more than two equations are produced, the correct two must be used. $\left(R = \frac{20mg}{13}, F = \frac{5mg}{13}\right)$
	$\Rightarrow \omega = \sqrt{\frac{16g}{13r}}$	A1*	Obtain given answer from correct working .
		[9]	
		(9)	
Alt1	Using N2L parallel and perpendicular to the incline. Perpendicular $R - mg\sin\theta = mr\omega^2\cos\theta$ Parallel $F + mg\cos\theta = mr\omega^2\sin\theta$	M1 A1A1 M1 A1A1	Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion. Note that the acceleration must have a sin/cos component. Accept any form of acceleration for the method mark only. Mark A's as above. A1A0 Unsimplified equation with at most one error A1A1 Correct unsimplified equation

5	 		Curve equation $x^2 + y^2 = r^2$
5a	Using x-axis $(\rho)\int x \times 2\sqrt{r^2 - x^2}\,\mathrm{d}x$ or Using y-axis $(\rho)\frac{1}{2}\int 2\left(\sqrt{r^2 - x^2}\right)^2 \mathrm{d}x$	M1	Use of correct integral. Limits not needed here. Accept an integral of the form: x-axis: $k\int x\sqrt{r^2 - x^2}\,\mathrm{d}x$ y-axis: $k\int r^2 - x^2\,\mathrm{d}x$
	x-axis $= -\frac{2}{3}(\rho)\left(r^2 - x^2\right)^{\frac{3}{2}}$ y-axis $= (\rho)\left(xr^2 - \frac{x^3}{3}\right)$	A1	Correct integration, ignore limits. Correct expression.
	$= \frac{2}{3}(\rho)r^3$	A1	Correct use of limits, 0 and r or $-r$ and r.
	Using x-axis $\frac{1}{2}\pi r^2 \rho \bar{x} = \rho\int_0^r 2xy\,\mathrm{d}x$ Using y-axis $\frac{1}{2}\pi r^2 \rho \bar{y} = \rho\frac{1}{2}\int_{-r}^{r} y^2\,\mathrm{d}x$ or $\frac{1}{2}\pi r^2 \rho \bar{y} = \rho\int_0^r y^2\,\mathrm{d}x$	M1	Complete method to obtain distance. Use of a correct formula, consistent with the axis and limits used, to find centre of mass with curve equation. ρ must appear on both sides or neither.
	$\bar{x} = \dfrac{\frac{2}{3}r^3}{\frac{1}{2}\pi r^2} = \dfrac{4r}{3\pi}$ *	A1*	Obtain given answer from correct working
ALT 1 5(a)	Parametric approach $x = r\cos\theta,\ \ y = r\sin\theta$		Curve equation $x^2 + y^2 = r^2$
	Using x-axis $2r^3\int_0^{\frac{\pi}{2}} \sin^2\theta\cos\theta\,\mathrm{d}\theta$	M1	Use of correct integral. Limits not needed here. Accept an integral of the form: $kr^3\int \sin^2\theta\cos\theta\,\mathrm{d}\theta$

<table>
<tr><td></td><td>$$= 2r^3\left[\frac{\sin^3\theta}{3}\right]_0^{\frac{\pi}{2}}$$</td><td>A1</td><td>Correct integration, ignore limits. Correct expression.</td></tr>
<tr><td></td><td>$$= \frac{2}{3}r^3$$</td><td>A1</td><td>Correct use of limits</td></tr>
<tr><td></td><td>$$\bar{x} = \frac{\frac{2}{3}r^3}{\frac{1}{2}\pi r^2}$$</td><td>M1</td><td>Complete method to obtain distance.
Use of correct formula.
ρ must appear on both sides or neither.</td></tr>
<tr><td></td><td>$$= \frac{4r}{3\pi} \ *$$</td><td>A1*</td><td>Obtain given answer from correct working</td></tr>
<tr><td></td><td></td><td>[5]</td><td></td></tr>
<tr><td>ALT 2 5(a)</td><td>Using y-axis
$$r^3\int_0^{\frac{\pi}{2}}\sin^3\theta \ \mathrm{d}\theta$$</td><td>M1</td><td>Use of correct integral. Limits not needed here.

Accept an integral of the form:
$$kr^3\int\sin^3\theta \ \mathrm{d}\theta$$</td></tr>
<tr><td></td><td>$$r^3\int_0^{\frac{\pi}{2}}\left(1-\cos^2\theta\right)\sin\theta \ \mathrm{d}\theta$$
$$= r^3\left[-\cos\theta + \frac{\cos^3\theta}{3}\right]_0^{\frac{\pi}{2}}$$</td><td>A1</td><td>Correct integration, ignore limits. Correct expression.</td></tr>
<tr><td></td><td>$$= \frac{2}{3}r^3$$</td><td>A1</td><td>Correct use of limits</td></tr>
<tr><td></td><td>$$\bar{x} = \frac{\frac{2}{3}r^3}{\frac{1}{2}\pi r^2}$$</td><td>M1</td><td>Complete method to obtain distance.
Use of correct formula.
ρ must appear on both sides or neither.</td></tr>
<tr><td></td><td>$$= \frac{4r}{3\pi} \ *$$</td><td>A1*</td><td>Obtain given answer from correct working</td></tr>
<tr><td></td><td></td><td>[5]</td><td></td></tr>
<tr><td>5b</td><td><table>
<tr><td></td><td>large</td><td>Small removed</td><td>Small added</td></tr>
<tr><td>mass</td><td>$8\pi a^2$</td><td>$2\pi a^2$</td><td>$2\pi a^2$</td></tr>
<tr><td>From AC</td><td>$\frac{16a}{3\pi}$</td><td>$\frac{8a}{3\pi}$</td><td>$(-)\frac{8a}{3\pi}$</td></tr>
</table></td><td>B1

B1</td><td>Correct mass ratios

Correct distances</td></tr>
<tr><td></td><td>Moments about AC</td><td>M1</td><td>All terms required. Dimensionally correct or equivalent for a parallel axis. Condone sign errors. If column vectors are used, this mark is awarded once the equation is written separate to the column vectors.</td></tr>
</table>

	$8\pi a^2\times\frac{16a}{3\pi}-2\pi a^2\times\frac{8a}{3\pi}-2\pi a^2\times\frac{8a}{3\pi}$ $=8\pi a^2 d$	A1	Correct unsimplified equation.
	$\frac{96a}{3\pi}=8d \;\Rightarrow d=\frac{4a}{\pi}$ *	A1*	Obtain given value from correct working. Need to see at least some simplification.
		[5]	
5c	Moments about perpendicular axis through *A*	M1	Dimensionally correct. Need all terms. Or equivalent for a parallel axis
	From *A* $4a\times 8\pi a^2-2a\times 2\pi a^2+6a\times 2\pi a^2=8\pi a^2\bar{x}$	A1ft A1ft	Unsimplified equation with at most one error. Correct unsimplified equation Follow their mass ratio
	$\Rightarrow \bar{x}=5a$	A1	Correct only. If measured from *B*, distance is *a*
	Correct use of trig to find an expression for $\tan\theta$	M1	$\tan\theta=\frac{d}{\bar{x}}$ or $\tan\theta=\frac{\bar{x}}{d}$ where $\bar{x}$ is distance from *A*.
	$\tan\theta=\frac{4}{5\pi}$	A1	Only
		[6]	
		(16)	

6a	In equilibrium	M1	Need all three forces. Dimensionally correct
	$mg+4mg\frac{l-e}{l}=4mg\frac{e}{l}$	A1 A1	Unsimplified equation with at most one error Correct unsimplified equation
	$5l=8e\Rightarrow e=\frac{5l}{8},$ $AE=l+\frac{5l}{8}=\frac{13l}{8}$ *	A1*	Obtain given answer from correct working. Must see *AE* =
ALT1	$mg+4mg\frac{(2l-AE)}{l}=4mg\frac{(AE-l)}{l}$	M1 A1 A1	Need all three forces. Dimensionally correct Unsimplified equation with at most one error Correct unsimplified equation
	$AE=\frac{13l}{8}$ *	A1*	Obtain given answer from correct working. Must see *AE* =
ALT2	$mg+4mg\frac{\left(\frac{l}{2}-e\right)}{l}=4mg\frac{\left(\frac{l}{2}+e\right)}{l}$	M1 A1 A1	Need all three forces. Dimensionally correct Unsimplified equation with at most one error Correct unsimplified equation
	$e=\frac{l}{8},\quad AE=l+\frac{l}{2}+\frac{l}{8}=\frac{13l}{8}$ *	A1*	Obtain given answer from correct working
		[4]	
6b	Equation of motion	M1	Need all terms. Dimensionally correct. Condone use of *a* for acceleration.
	$4mg\frac{\frac{5l}{8}+x}{l}-4mg\frac{\frac{3l}{8}-x}{l}-mg=-m\ddot{x}$	A1 A1	Unsimplified equation with at most one error. Correct unsimplified equation Note: the question states *x* is measured vertically down.
	$\Rightarrow -m\ddot{x}=\frac{8mg}{l}x,\ \ddot{x}=-\frac{8g}{l}x$ *	A1*	Obtain given answer from correct working. Must use $\ddot{x}$
		[4]	
6c	Use of $v^2=\omega^2\left(a^2-x^2\right)$ with $a=\frac{3l}{8}$	M1	Or use of equivalent correct formula
	$=\frac{8g}{l}\left(\frac{9}{64}l^2-\frac{1}{64}l^2\right)$	A1	Correct unsimplified expression for v or v^2
	$v=\sqrt{gl}$	A1	Correct only
		[3]	

6d	$x = \frac{3l}{8}\cos\omega t$	B1	Use of relevant formula with correct amplitude $x = \frac{3l}{8}\cos\omega t$ or $x = \frac{3l}{8}\sin\omega t$
	Use of $-\frac{l}{8} = \frac{3l}{8}\cos\omega t$ or $\frac{l}{8} = \frac{3l}{8}\sin\omega t$ and correct use of $\frac{1}{2}\times\frac{2\pi}{\omega}$ or $\frac{l}{8} = \frac{3l}{8}\cos\omega t$ and correct use of $\pi - \cos^{-1}\left(\frac{1}{3}\right)$	M1	Complete method to find t or required time $t = \frac{1}{\omega}\cos^{-1}\left(-\frac{1}{3}\right)$ or $t = \frac{1}{\omega}\sin^{-1}\left(\frac{1}{3}\right)$ with $\frac{1}{2}$period or $t = \frac{1}{\omega}\cos^{-1}\left(\frac{1}{3}\right)$ with π
	Required time $\frac{2}{\omega}\cos^{-1}\left(\frac{-1}{3}\right) = \sqrt{\frac{l}{2g}}\cos^{-1}\left(\frac{-1}{3}\right)$ or $\frac{\pi}{\omega} + \frac{2}{\omega}\sin^{-1}\left(\frac{1}{3}\right) = \sqrt{\frac{l}{8g}}\left(\pi + 2\sin^{-1}\left(\frac{1}{3}\right)\right)$ or $\frac{2}{\omega}\left[\pi - \cos^{-1}\left(\frac{1}{3}\right)\right] = \sqrt{\frac{l}{2g}}\left[\pi - \cos^{-1}\left(\frac{1}{3}\right)\right]$	A1	Or equivalent, accept $1.91\sqrt{\frac{l}{2g}},\ 1.35\sqrt{\frac{l}{g}},\ 0.43\sqrt{l}$ $3.82\sqrt{\frac{l}{8g}}$ $\cos^{-1}\left(\frac{-1}{3}\right) = 1.91...$
		[3]	
		(14)	

7a	Conservation of mechanical energy:	M1	All terms required. Dimensionally correct $\cos\theta = \frac{5}{13}, \sin\theta = \frac{12}{13}$
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\left(r + r\cos\theta\right)$	A1	Correct unsimplified equation
	$v^2 = u^2 - \frac{36}{13}gr$ *	A1*	Obtain given answer from correct working
		[3]	
7b	Equation of motion	M1	All terms required. Dimensionally correct. Condone sign errors and sin/cos confusion. Condone use of $R = 0$
	$R + mg\cos\theta = \frac{mv^2}{r}$	A1	Correct unsimplified equation. Condone (strict) inequality the right way round.
	Use $R \ldots 0$ and solve for u^2	M1	Complete method to obtain u^2 Condone use of $R = 0$ or $R > 0$
	$\frac{mv^2}{r} - mg\cos\theta \ldots 0$ $\Rightarrow u^2 - \frac{36}{13}gr \ldots \frac{5}{13}gr, \quad u^2 \ldots \frac{41}{13}gr$ *	A1*	Obtain given answer from correct working. Must have stated the inequality $R \geq 0$ If there is no reference to R, the max mark in (b) is M1A1M1A0*
		[4]	
7c	$BC = 2r\sin\theta = \frac{24}{13}r$	B1	Or equivalent $BC = 1.846\ldots r$
	Relevant vertical motion Eg time to return to the level of BC	M1	Complete method vertically using *suvat*
	$t = \frac{2v\sin\theta}{g} = \frac{24v}{13g}$	A1	Correct unsimplified expression for time Accept $\frac{24}{13g} \times 4\sqrt{\frac{gr}{13}}$, $\frac{24}{13}\sqrt{\frac{16r}{13g}}$ $\frac{96}{13}\sqrt{\frac{r}{13g}}$, $0.65\sqrt{r}$
	Relevant horizontal motion Eg distance travelled by P	M1	Complete method horizontally
	$= \left(v\cos\theta\right)t = v^2 \times \frac{120}{169g}$	A1	Correct unsimplified expression for distance $0.87r$, $\frac{1920}{2197}r$, $0.0892gr$
	$= \frac{16gr}{13} \times \frac{120}{169g} = \frac{160r}{169} \times \frac{12}{13} < 2r \times \frac{12}{13}$	A1*	Obtain given conclusion from correct working

	hence falls into the bowl *		
ALT 1 for last 3 marks	Horizontal: time, T, required to travel the length BC	M1	Complete method horizontally
	$2r\sin\theta = v\cos\theta \times T$ $T = \dfrac{2r\frac{12}{13}}{4\sqrt{\frac{gr}{13}} \times \frac{5}{13}} = 1.38\sqrt{r}$	A1	Correct unsimplified expression for T
	$t < T$ since $0.654\sqrt{r} < 1.38\sqrt{r}$ hence falls into the bowl *	A1*	Obtain given conclusion from correct working
ALT 2 for last 3 marks	Horizontal: speed, V, required to reach C	M1	Complete method horizontally
	$-V\sin\theta = V\sin\theta - g\dfrac{2r\sin\theta}{V\cos\theta}$ $\Rightarrow V = \sqrt{\dfrac{gr}{\cos\theta}} = \sqrt{\dfrac{13gr}{5}}$	A1	Correct unsimplified expression for V
	$v < V$ since $\sqrt{\dfrac{13gr}{5}} < \sqrt{\dfrac{16gr}{13}}$ hence falls into the bowl *	A1*	Obtain given conclusion from correct working
	SC: If range formula is quoted correctly award M1A1M1A1. Range = $\dfrac{2v^2\sin\theta\cos\theta}{g}$		
		[6]	
		(13)	